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Questions and Answers

Concerning Balakrischnan's paradox [*J. Statistical Phys.* **1**: 227 (1969)] the following remark seems pertinent.

The condition

$$\frac{1}{\Delta}\left[\int_t^{t+\Delta} n(\zeta)\,d\zeta\right]^2 \to 1 \qquad \text{as} \quad \Delta \to 0 \tag{1}$$

that is (assuming $\Delta > 0$ for convenience), $|\int_t^{t+\Delta} n(\zeta)\,d\zeta| \sim \Delta^{1/2}$, implies that

$$\frac{|\int_0^{t+\Delta} n(\zeta)\,d\zeta - \int_0^t n(\zeta)\,d\zeta|}{\Delta} \sim \Delta^{-1/2}$$

Hence, if condition (1) holds, the function

$$N(t) = N(0) + \int_0^t n(\zeta)\,d\zeta$$

is not differentiable in the ordinary sense. On the other hand, it is natural to define

$$\frac{d}{dt} N(t) = n(t)$$

Then, if

$$\begin{aligned} x(t) &= \exp\left[\int_0^t n(\zeta)\,d\zeta\right] \\ \frac{d}{dt} x(t) &= x(t)\,n(t) \end{aligned} \tag{2}$$

by definition. However, manipulations of Eq. (2) must take into account the condition (1), and thus may not be those of ordinary calculus.

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